On The Metaphysics Of Savits

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CONTENTS

Chapter 1

DICE

1.1 Basics of Dice

1.1.1 Distribution of Successes

For further analysis it will be useful to have a distribution function for margins of success.

To obtain such a distribution we start by observing that each roll can be divided into four groups: critical successes, successes, failures and critical failures, as shown in Eq. (1.1). Further, this tuple has a multinomial distribution

$$(C_s, S, F, C_f) \sim \mathbf{MN}\left(d, 4, p\right) \tag{1.1}$$

$$p = \left[1/12, \frac{12 - \mathbf{DT} - 1}{12}, \frac{\mathbf{DT} - 1}{12}, \frac{1}{12}\right] \tag{1.2}$$

We can easily map a tuple to a success margin:

$$M = 2C_s + S - F - 2C_f \tag{1.3}$$

Therefore, our distribution for M is a sum over all tuples that produce the correct m

$$Pr(M = m) = \sum_{C_s, S, F, C_f; M=m} \mathbf{MN}\left(d, 4, p\right) \tag{1.4}$$

Instead of trying all combinations of variables and summing the correct ones, we can use a system of Diophantine equations to reduce the search space.

We start with two equations

$$2c_s + s - f - 2c_f = m \tag{1.5}$$

$$c_s + s + f + c_f = d \tag{1.6}$$

which resolve into a two-dimensional system

$$c_s = m - d + 2f + 3c_f \tag{1.7}$$

$$s = -m + 2d - 3f - 4c_f \tag{1.8}$$

Next, we use inequalities $c_s \geq 0, s \geq 0$ to constrain the space for c_f, based on f. An example plot in Fig. 1.1 graphically shows the area defined by these equations and inequalities for one combination of d and m.

$$\frac{m - d + 2f}{-3} \leq c_f \leq \frac{-m + 2d - 3f}{4} \tag{1.9}$$

Because the lower bound is not necessarily greater than 0, we will also use $c_f \geq 0$.

We can now also constrain f by observing that the lower bound for c_f can cross $c_f = 0$ before $f = d$.

$$0 \leq f \leq \frac{2d - m}{3} \tag{1.10}$$

Putting the sum together, and adding floors and ceilings to limit the sum to integers, we get:

$$Pr(M=m) = \sum_{f=0}^{\lfloor \frac{2d-m}{3} \rfloor} \sum_{c_f=\max(0,\lceil \frac{m-d+2f}{-3} \rceil)}^{\lfloor \frac{-m+2d-3f}{4} \rfloor} \quad (1.11)$$

$$\frac{d!}{c_s!s!f!c_f!} \frac{n_s^s n_f^f}{12^d} \quad (1.12)$$

$$n_f = \mathbf{DT} - 1 \quad (1.13)$$

$$n_s = 12 - n_s \quad (1.14)$$

While this equation is not very illuminating, it does allow us to efficiently compute the distribution and thus answer various questions regarding the results of dice rolls.

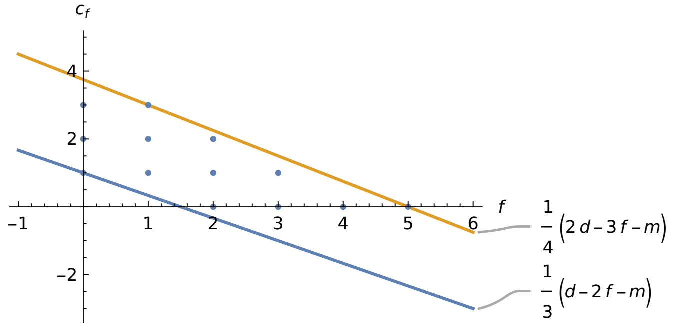

Figure 1.1: Example of candidate combinations of f and c_f with upper and lower bounds for c_f shown. In this plot, $d = 12$ and $m = 8$.

1.1.2 Probability of Success

The first question to ask is: "What is the probability of success, given a fixed **DT**?"

We will use the binomial distribution $\mathrm{B}(20, 7/20)$ for the size of the dice pool, to compute the expectation. This is a fairly arbitrary choice that assumes 7 is the average pool size. However, it allows us to analyze the probabilities regardless of what the actual current size of the pool is.

Table 1.1 shows the probability of success for a given **DT** as well as number of successes in a number of attempts.

1.1.3 Margin of Success

We also want to know what the expected margin of success is, given that the player succeeded in the roll, $\mathrm{E}[M|M \geq 0]$. Table 1.2 shows the expectations.

Of note are the expected numbers of successes for **DT** 11. The requirement for critical successes there causes a large variance in results. It is also important to remember that this does not show the full range of results. The highest possible result is $2d$, which for one die is the only possible (successful) result, while other numbers of dice have wider ranges.

DT	p	Succeeds		
1	98%	51	in	52
2	95%	20	in	21
3	90%	8	in	9
4	82%	4	in	5
5	70%	2	in	3
6	56%	1	in	2
7	42%	2	in	5
8	28%	1	in	4
9	17%	1	in	6
10	9%	1	in	12
11	4%	1	in	26

Table 1.1: Expected chance of success.

The variance on the upper half of the **DT** range is very low. Even large dice pools can only expect 1 or 2 successes. With a drop in **DT** the dice pool really comes into its own and can be expected to produce more successes. This means that building a large dice pool is not enough to ensure a large number of successes. The player must actually reduce the **DT** to get a high likelihood of more successes.

	Difficulty Threshold										
d	1	2	3	4	5	6	7	8	9	10	11
1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.2	1.3	1.5	2.0
2	2.1	1.8	1.5	1.3	1.1	1.0	0.8	0.7	0.7	0.7	1.0
3	2.6	2.3	2.1	1.9	1.8	1.6	1.5	1.3	1.1	0.7	0.3
4	3.5	3.1	2.7	2.3	1.9	1.6	1.4	1.2	1.1	1.2	2.0
5	4.4	3.7	3.1	2.7	2.3	2.0	1.7	1.4	1.1	0.9	1.1
6	5.1	4.3	3.6	3.0	2.5	2.1	1.7	1.5	1.3	1.1	0.5
7	6.0	5.0	4.1	3.4	2.8	2.3	2.0	1.6	1.3	1.2	1.9
8	6.8	5.6	4.6	3.7	3.0	2.5	2.0	1.7	1.4	1.1	1.0
9	7.5	6.2	5.1	4.1	3.3	2.7	2.2	1.8	1.4	1.2	0.6
10	8.4	6.9	5.5	4.4	3.5	2.8	2.2	1.8	1.5	1.2	1.8
11	9.2	7.5	6.0	4.8	3.7	3.0	2.4	1.9	1.5	1.2	1.0
12	10.0	8.2	6.5	5.1	4.0	3.1	2.4	1.9	1.6	1.3	0.7

Table 1.2: Expected number of successes, given the test was successful.

Further, we may want to know more about the distribution of successes. Once again employing the distribution of dice pools from Section 1.1.2 we can compute the quantiles (for $[0.1, 0.25, 0.5, 0.75, 0.9]$) in Table 1.4. Note that this is still conditioned on the whole roll being successful, i.e., we are considering only non-negative margins of success.

Like with expectations, there is an anomaly in **DT** 11, because only critical successes pass that **DT**. Another thing worth noting is that for **DT** 8–11, the median number of successes is 1. This means that even if the roll is successful, the margin is likely as not at most 1.

d	Difficulty Threshold 1	2	3	4	5	6	7	8	9	10	11
1	1.0	0.9	0.8	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.2
2	1.8	1.5	1.3	1.0	0.8	0.7	0.5	0.4	0.3	0.2	0.2
3	2.6	2.2	1.8	1.4	1.1	0.9	0.6	0.4	0.3	0.2	0.1
4	3.4	2.8	2.3	1.8	1.3	1.0	0.7	0.4	0.3	0.2	0.1
5	4.2	3.5	2.7	2.1	1.6	1.1	0.7	0.4	0.2	0.1	0.1
6	5.0	4.1	3.2	2.4	1.8	1.2	0.8	0.4	0.2	0.1	0.0
7	5.9	4.8	3.7	2.8	2.0	1.3	0.8	0.4	0.2	0.1	0.0
8	6.7	5.4	4.2	3.1	2.1	1.4	0.8	0.4	0.2	0.1	0.0
9	7.5	6.1	4.7	3.4	2.3	1.5	0.8	0.4	0.2	0.1	0.0
10	8.3	6.7	5.2	3.7	2.5	1.5	0.8	0.4	0.2	0.1	0.0
11	9.2	7.4	5.6	4.1	2.7	1.6	0.9	0.4	0.1	0.0	0.0
12	10.0	8.0	6.1	4.4	2.9	1.7	0.9	0.4	0.1	0.0	0.0

Table 1.3: Expected number of successes overall.

DT	q_{10}	q_{25}	q_{50}	q_{75}	q_{90}
1	3.1	4.3	6.6	7.9	8.1
2	1.7	3.3	5.1	6.8	8.0
3	0.8	2.4	4.2	5.8	7.4
4	0.4	1.6	3.3	5.0	6.6
5	0.0	0.9	2.6	4.2	5.7
6	0.0	0.7	1.9	3.6	5.0
7	0.0	0.4	1.6	2.9	4.5
8	0.0	0.0	1.0	2.4	3.9
9	0.0	0.0	1.0	2.0	3.4
10	0.0	0.0	0.8	1.8	2.9
11	0.3	0.5	1.0	1.0	2.4

Table 1.4: Quantiles of the number of successes for a given **DT**.

1.1.4 Threshold–Margin Combinations

It is useful to know how to give players a challenge with a certain chance of success. There is no simple formula for this, but in Table 1.5 we show, for 10% increments, what combinations of **DT** and margin give a given chance of passing the test. We are using the same distribution for dice pool sizes as in Section 1.1.2.

We separated Table 1.5 into what we might call “difficulty classes”, denoted by an intuitive chance of success. A quick glance at the numbers reveals that moving from once class to the next requires adding 1 to the **DT** or adding 1–2 to the required margin.

Succeeds	DT	m
19 in 20	1	0–1
	2	0
9 in 10	1	2–3
	2	1–2
	3	0–1
4 in 5	1	4–5
	2	3–4
	3	2
	4	0–1
	5	0
1 in 2	1	6–7
	2	5
	3	3–4
	4	2–3
	5	1–2
	6	0–1
	7	0

Succeeds	DT	m
1 in 3	1	8
	2	6–7
	3	5–6
	4	4–5
	5	3–4
	6	2–3
	7	1–2
	8	0
1 in 10	1	9–10
	2	8–9
	3	7–8
	4	6–7
	5	5–6
	6	4–5
	7	3
	8	1–2
	9	0–1
	10	0

Succeeds	DT	m
1 in 20	1	11–12
	2	10–11
	3	9 –10
	4	8–9
	5	7–8
	6	6–7
	7	4–5
	8	3–4
	9	2–3
	10	1–2
	11	0
1 in 100	1	13
	2	12
	3	11
	4	10
	5	9
	6	8
	7	6
	8	5
	9	4
	10	3
	11	1

Table 1.5: Combinations of **DT** and margin that give a certain chance of success.

1.2 Combat Pool Steady State

The combat pool starts with the proficiency stat, i.e., **STR** or **DEX**. However, after rolling, the change in pool size is determined by **CON**. After a number of turns, the expected size of the pool should converge on a number $\mathrm{E}[d]$.

We start by assuming that $\mathrm{E}[d]$ is a number such that

$$\mathrm{E}[d_{n+1}] = \mathrm{E}[d_n] \tag{1.15}$$

where n is the turn after which the steady state is achieved.

To compute $\mathrm{E}[d_n]$ for any n we can use

$$\mathrm{E}[d_n] = \mathrm{E}[d_{n-1}] - p_f \cdot \mathrm{E}[d_{n-1}] + \mathbf{CON} \tag{1.16}$$

where $p_f = \mathbf{DT}/12$ is the probability of failure.

Using assumptions in Eqs. (1.15) and (1.16) we write the following equations:

$$\mathrm{E}[d] = \mathrm{E}[d] - p_f \cdot \mathrm{E}[d] + \mathbf{CON} \tag{1.17}$$

$$p_f \cdot \mathrm{E}[d] = \mathbf{CON} \tag{1.18}$$

$$\mathrm{E}[d] = \frac{\mathbf{CON}}{p_f} \tag{1.19}$$

As p_f approaches 1, the steady state becomes just **CON**, meaning the dice pool is entirely discarded and refreshed by **CON** on every turn. On the other hand, as p_f approaches 0, the expected dice pool size increases. Specifically at 0, it shoots off toward infinity, as the dice pool loses no dice and **CON** dice get added to it on every turn.

Chapter 2
CLASSES

Basic Class Structure

Combat

Stats Each class grants +1 in three separate stats; no two classes grant the exact same stats.

Most combat classes grant bonuses to physical stats, aiming for the class to be self-sufficient, such as the warrior group. Some classes, such as rangers, add mental stats, to support other classes.

Standard Weapons The standard weapons are purely for flavor. They act as a guide to how that class should feel.

Weapon Proficiencies Weapon proficiencies are chosen such that they include the standard weapons.

Armor proficiencies Armor proficiencies are also selected to support the flavor of the class. It should reflect how vulnerable the class should feel, taking into account how the stats, i.e., **STB** and **DEX** compensate for any missing armor.

Note that every class can wear no armor, e.g., normal clothes.

Skills The number of skills should roughly reflect the mental stats a class grants. More support-focused classes should grant more skills.

Abilities Each class group has one ability that all classes in that group share. Other than that, each class should have at least 4 abilities.

The abilities are what really differentiates one class from another.

The number of passive and reactive abilities affects how passive the class feels. Something like a guard should feel more like guarding – standing still and reacting to what other people do.

Chapter 3
ABILITIES

3.1 Fighter

Sure Strike Sure Strike is an attack that guarantees a hit, but is not as good as a normal attack is expected to be.

Assuming the average **DT** in a fight is 6, with **CON** 3, the expected size of a dice pool is 7. With a dice pool that size, the expected number of successes is around 2–3.

If we make the player sacrifice d' dice to get $d'/2$ successes it means that with 7 dice, the player could get 3 successes, which is more than expected. On the other hand, they would have to sacrifice all 7 dice, instead of 3–4 they would likely lose to failures.

If, instead, they choose to spend 3–4 dice, they get 1–2 successes, which is fewer than expected.

Sweep Sweep hits multiple enemies. For flavor reasons, this ability should be limited by the player's **STR**, i.e., the character must be strong to be able to push their weapon through multiple opponents.

Hard Strike Hard Strike should be more powerful than Sure Strike, bur can be less reliable. Using the same assumption of **DT** 6 and a pool of 7, if we assume a **STR** of 3, adding **STR** to successes roughly doubles the number of successes.

There must be a cost the prevents the player using this ability on every turn. Discarding the entire dice pool would mean it would likely take two turns to build it up to the previous size, which is congruent with the doubling of the damage. However, discarding a dice pool of 1–3 on every turn to get 3 successes would be too cheap. Therefore, a minimum must be set. Requiring $2 \cdot$ **STR** dice to be discarded seems like a good balance. Making that number the maximum also avoids a problem with large dice pools. Even though statistically speaking you can expect the same number of successes from any dice pool, discarding 10 dice would feel worse than discarding 6.

Parry The basic design of Parry is to make a roll and remove the number of defender's successes from that of the attacker.

Being about to use this ability on every turn would be too powerful and too tedious. Therefore, it must be limited to occasional use, similar to Hard Strike.

3.2 Rogue

Disappear Disappear is intended to be used in combat (cf. Camouflage). However, any character paying attention should be able to spot a thief trying to disappear.

This is modeled by giving characters that attacked the thief and characters that did not attack anyone – and are therefore free to pay attention – to make an opposed roll.

3.3 Ranger

Camouflage Camouflage is supposed to be used before an encounter. Therefore, camouflage does not work on characters that are paying attention.

To balance that, camouflage allows movement. While a Thief is supposed to disappear and reappear multiple times in an encounter, a hunter is supposed to hide *before* the encounter and strike when the time is right.

Chapter 4
OPPONENTS

4.0.1 Drub

Drubs are vaguely modeled on goblins, but represent a more feral variant.

Drubs have negative **CHA**, representing their ugliness, and negative **CMP**, indicating that they are quick to anger. This puts their **MP** in the negative: they cannot be reasoned with.

Their high **DEX** represents jumpy and nervous nature. Average **STB**, because, while small, they are still used to fighting.

4.0.2 Bandit

Bandit is a lightweight brawler.

4.0.3 Horned Bear

Horned bear is a heavyweight brawler.

She is a harder enemy and therefore has proficiency 2. She has regular intelligence or charisma, but she has negative knowledge (she is a bear) and a bit of composure (she knows what she's about.)